

ASYMPTOTES OF RATIONAL FUNCTIONS

$$y = f(x) = \frac{N(x)}{D(x)}$$

where $N(x)$ and $D(x)$ are polynomials

HORIZONTAL ASYMPTOTES, $y = b$

A horizontal asymptote is a horizontal line that is not part of a graph of a function but guides it for x -values “far” to the right and/or “far” to the left. The graph may cross it but eventually, for large enough or small enough values of x (approaching $\pm\infty$), the graph would get closer and closer to the asymptote without touching it. A horizontal asymptote is a special case of a slant asymptote.

A ”recipe” for finding a horizontal asymptote of a rational function:

Let

deg $N(x)$ = the degree of a numerator and **deg $D(x)$** = the degree of a denominator.

deg $N(x) = \text{deg } D(x)$	deg $N(x) < \text{deg } D(x)$	deg $N(x) > \text{deg } D(x)$
$y = \frac{\text{leading coefficient t of } N(x)}{\text{leading coefficient t of } D(x)}$	$y = 0$ which is the x -axis	There is no horizontal asymptote.

Another way of finding a horizontal asymptote of a rational function:

Divide $N(x)$ by $D(x)$. If the quotient is constant, then $y =$ this constant is the equation of a horizontal asymptote.

Examples

Ex. 1

$$y = \frac{-2x^3 - 3x + 5}{x^3 + 1} = -2 + \frac{-3x + 7}{x^3 + 1}$$

HA: $y = -2$

because $\frac{-3x + 7}{x^3 + 1}$ approaches 0 as x increases.

Ex. 2

$$y = \frac{2x + 1}{x} = 2 + \frac{1}{x}$$

HA : $y = 2$

because $\frac{1}{x}$ approaches 0 as x increases.

Ex. 3

$$y = \frac{3x^2}{x+1} = (3x - 3) + \frac{3}{x+1} \text{ approaches } \infty \text{ as } x \text{ increases (} y = 3x - 3 \text{ is a slant asymptote.)}$$

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SLANT (OBLIQUE) ASYMPTOTE, $y = mx + b, m \neq 0$

A slant asymptote, just like a horizontal asymptote, guides the graph of a function only when x is close to $\pm\infty$ but it is a slanted line, i.e. neither vertical nor horizontal. A rational function has a slant asymptote if the degree of a numerator polynomial is 1 more than the degree of the denominator polynomial.

A "recipe" for finding a slant asymptote of a rational function:

Divide the numerator $N(x)$ by the denominator $D(x)$. Use long division of polynomials or, in case of $D(x)$ being of the form: $(x - c)$, you can use synthetic division.

The equation of the asymptote is $y = mx + b$ which is the quotient of the polynomial division (ignore remainder)

Examples

$$f(x) = \frac{6x^3 - 1}{-2x^2 + 18}$$

deg $N(x) = 3$, deg $D(x) = 2$.

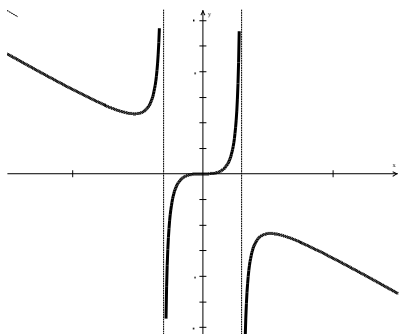
Perform long division $D(x) \overline{)N(x)}$:

$$\begin{array}{r} -3x \\ -2x^2 + 18 \overline{) 6x^3 \\ \underline{-6x^3 + 54x} \\ 54x - 1 \end{array} \leftarrow \text{this is the remainder}$$

$$f(x) = -3x + \frac{54x - 1}{-2x^2 + 18}$$

The slant asymptote's equation is:

$$\boxed{y = -3x}$$



$$f(x) = \frac{2x^2 + x - 5}{x + 1}$$

deg $N(x) = 2$, deg $D(x) = 1$.

Perform synthetic division:

$$\begin{array}{r|rrrr} -1 & 2 & 1 & -5 & \\ & \downarrow & -2 & 1 & \\ \hline & 2 & -1 & -4 & \end{array}$$

Zero of the denominator $\boxed{-1}$ $\boxed{-4}$ ← this is the remainder

$$f(x) = 2x - 1 + \frac{-4}{x + 1}$$

The slant asymptote's equation is:

$$\boxed{y = 2x - 1}$$

