ASYMPTOTES OF RATIONAL FUNCTIONS

$$y = f(x) = \frac{N(x)}{D(x)}$$

where N(x) and D(x) are polynomials

HORIZONTAL ASYMPTOTES, y = b

A horizontal asymptote is a horizontal line that is not part of a graph of a function but guides it for x-values "far" to the right and/or "far" to the left. The graph may cross it but eventually, for large enough or small enough values of x (approaching $\pm\infty$), the graph would get closer and closer to the asymptote without touching it. A horizontal asymptote is a special case of a slant asymptote.

A "recipe" for finding a horizontal asymptote of a rational function:

Let

deg N(x) = the degree of a numerator and **deg D(x)** = the degree of a denominator.

deg N(x) = deg D(x)	deg N(x) < deg D(x)	deg N(x) > deg D(x)
$y = \frac{\text{leading coefficient of } N(x)}{\text{leading coefficient of } D(x)}$	$y = 0$ which is the x-axis	There is **no** horizontal asymptote.

Another way of finding a horizontal asymptote of a rational function:

Divide N(x) by D(x). If the quotient is constant, then y = this constant is the equation of a horizontal asymptote.

Examples

Ex. 1

$$\mathbf{y = \frac{-2x^3-3x+5}{x^3+1} = -2 + \frac{-3x+7}{x^3+1}}$$

HA: $\boldsymbol{y = -2}$

because $\frac{-3x+7}{x^3+1}$ approaches 0 as x increases.

Ex. 2

$$\mathbf{y = \frac{2x+1}{x} = 2 + \frac{1}{x}}$$

HA : $\boldsymbol{y = 2}$

because $\frac{1}{x}$ approaches 0 as x increases.

Ex. 3

$\boldsymbol{y = \frac{3x^2}{x+1} = (3x-3) + \frac{3}{x+1}}$ approaches ∞ as x increases (y = 3x – 3 is a slant asymptote.)

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SLANT (OBLIQUE) ASYMPTOTE, $y = mx + b, m \neq 0$

A slant asymptote, just like a horizontal asymptote, guides the graph of a function only when x is close to $\pm\infty$ but it is a slanted line, i.e. neither vertical nor horizontal. A rational function has a slant asymptote if the degree of a numerator polynomial is 1 more than the degree of the denominator polynomial.

A "recipe" for finding a slant asymptote of a rational function:

Divide the numerator N(x) by the denominator D(x). Use long division of polynomials or, in case of D(x) being of the form: $(x-c)$, you can use synthetic division.

The equation of the asymptote is $y = mx + b$ which is the quotient of the polynomial division (ignore remainder)

Examples

$$f(x) = \frac{6x^3 - 1}{-2x^2 + 18}$$

deg N(x) = 3, deg D(x) = 2.

Perform long division $D(x)\overline{)N(x)}$:

$$\begin{array}{r} -3x \\ -2x^2 + 18 \overline{) 6x^3 \qquad -1} \\ \underline{-6x^3 + 54x} \\ 54x - 1 \end{array}$$

$54x - 1 \leftarrow$ this is the remainder

$$f(x) = -3x + \frac{54x - 1}{-2x^2 + 18}$$

The slant asymptote's equation is:

$$\boxed{y = -3x}$$

$$f(x) = \frac{2x^2 + x - 5}{x + 1}$$

deg N(x) = 2, deg D(x) = 1.

Perform synthetic division:

Zero of the denominator

-1	**2**	**1**	**-5**
		-2	**1**
	2	**-1**	**-4**

-4 ← this is the remainder

$$f(x) = 2x - 1 + \frac{-4}{x + 1}$$

The slant asymptote's equation is:

$$\boxed{y = 2x - 1}$$