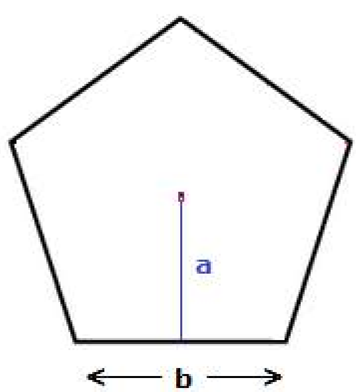
Area of a Pentagon, Regular

Although a pentagon only has 1 more extra side/edge than a square or a rectangle.

Working out the area of a regular Pentagon, is slightly more involved and complex in comparison.

Pentagon Area Formula

If you have a regular Pentagon of the form:



The area can be worked out by $\frac{5}{2} \times \mathbf{a} \times \mathbf{b}$.

The more complex work usually involves working out the value of the apothem line **a** first.

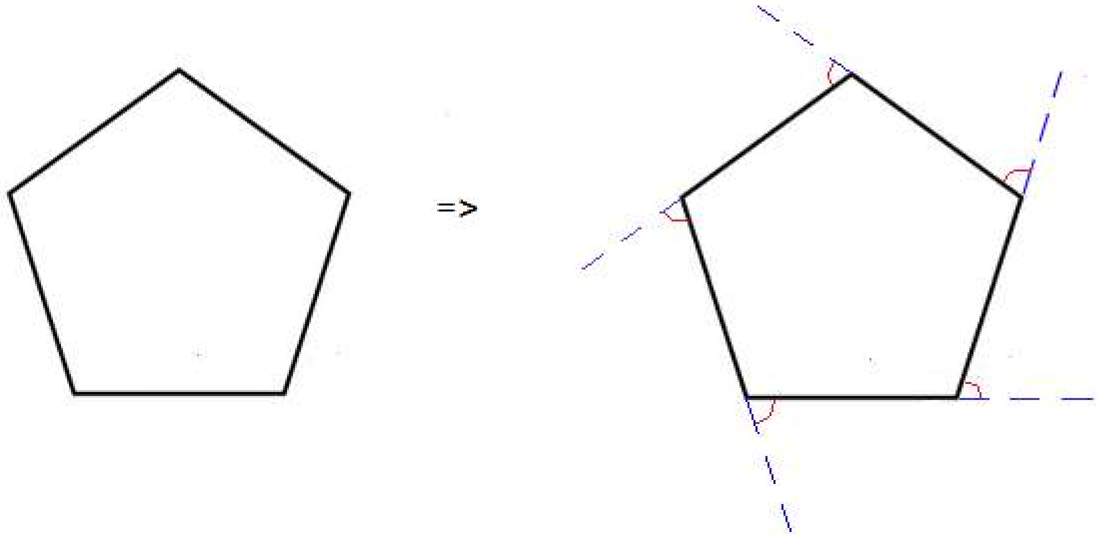
The majority of this page shows where this formula comes from, and how to work out the length of the apothem line **a**.

But if you're just looking for an example on its own, there is one at the bottom of this page, where you can scroll straight to if you wish.

Derive Area of a Pentagon Formula

Pentagon Interior and Exterior Angles

For a regular Pentagon, all *exterior angles* add up to **360°**.



As can be seen, there are 5 red exterior angles with a Pentagon, and they are all the same size.

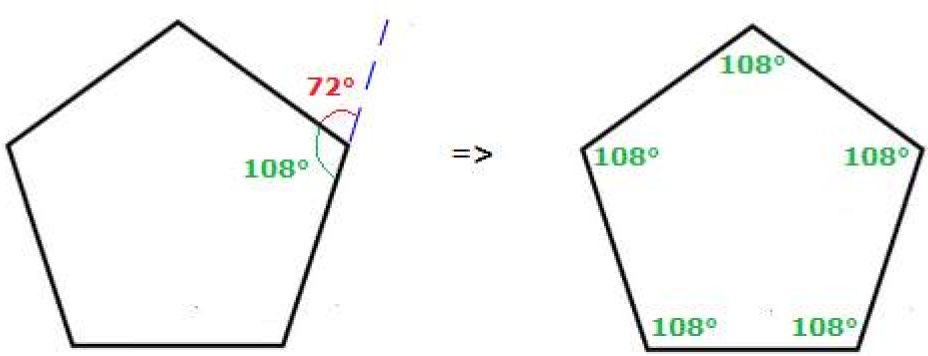
Regardless of how big or small the Pentagon itself is.

So the size of 1 exterior angle is $\frac{360^{\circ}}{5}$ = **72°**.

This fact can tell us what size the interior angles are.

As one side of a straight line is **180°**.

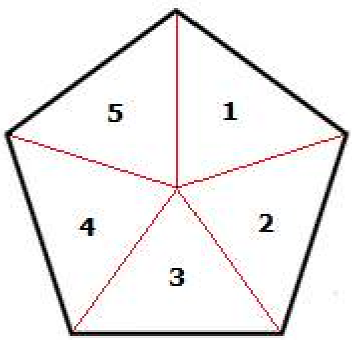
The size of a pentagon interior angle can be given by **180°** − **72°** = **108°**



Split Pentagon into Triangles

Now leaving the size of the interior angles to the side for a moment.

A regular Pentagon can be split into **5** equal sized triangles, by drawing straight lines from the center of the Pentagon, to the corners.



As all the triangles are the same size, if we can find the area of just **1** triangle.

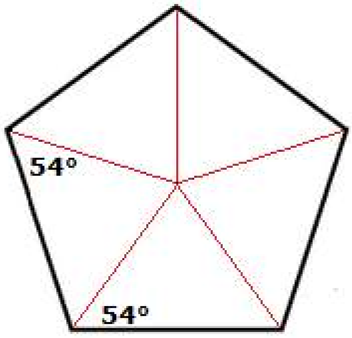
We can then find the area of a Pentagon in its entirety, by multiplying the area of the triangle by **5**.

To find the area of 1 of the triangles though, a bit more information is needed than what we have thus far.

Each of the straight lines from the center of the Pentagon to the corners, actually cut each corner in half evenly.

As each interior angle is **108°**.
The size of each angle on each side of the straight lines is given by:

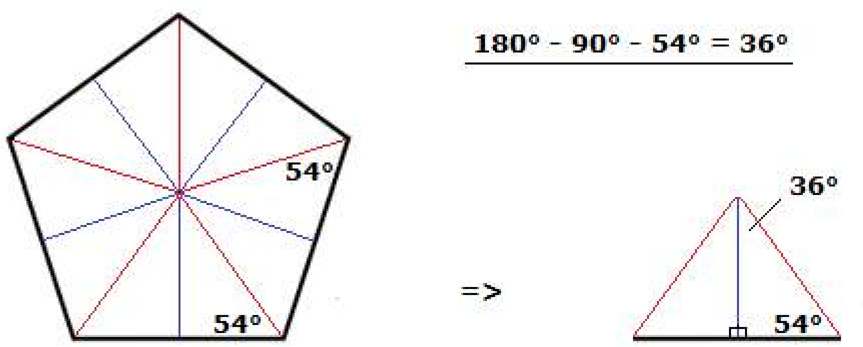
$\frac{108^{o}}{2}$ = **54°**.



Further splitting of the Triangles

Now the next thing to consider, is that each of the **5** triangles inside the Pentagon, can themselves be split into **2** smaller right angle triangles.

With each right angle triangle having 3 interior angles that add up to **180°**.

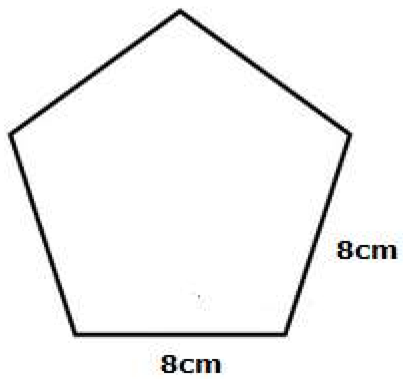


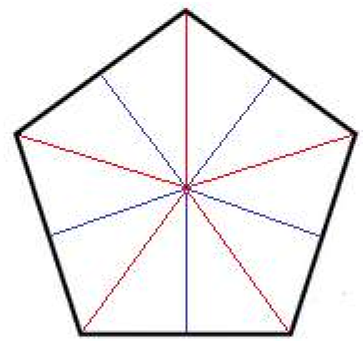
So each one of these new smaller triangles, is a Right Angle Triangle.

A Right Angle Triangle, of which we know the size of all the angles.

Now lets' see how all this helps with working out the area of a specific regular Pentagon, where the sides/edges are all **8cm** in length.

Working out Pentagon Area



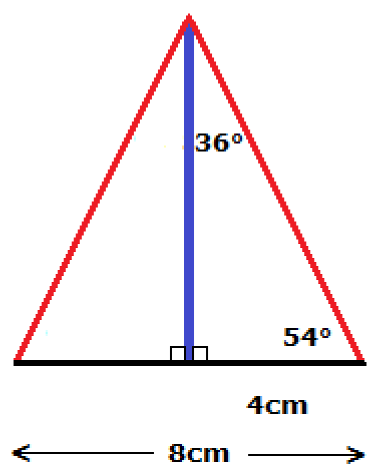


Like any standard regular Pentagon, this Pentagon can be split up into the triangles seen earlier.

The angles of the smaller right angle triangle, are always the same size, no matter how big or small the Pentagon.

But the length of the sides can differ.

With this Pentagon, the base of one of the smaller right angle triangles will be **4cm**, half of a whole **8cm** edge.

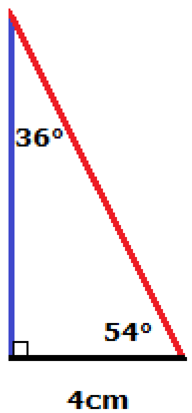


As the area of a triangle is given by $\frac{1}{2}$ × **BASE** × **HEIGHT**.

We just need to work out the length of the height, which is the blue line.
Then we can work out the area of the whole larger triangle with base **8cm**.

Then, this triangle area multiplied by **5**, will give the area of the whole Pentagon.

Concentrating on just one of the right angle triangles, we can use some Trigonometry to work out the size of the blue line, the height.



$$\textit{tan} = \frac{\text{opposite}}{\text{adjacent}}$$

For angle **54°** here, the "*adjacent*" side is **4cm**, and the "*opposite*" side is the height.

$$\textit{tan}(54°) = \frac{height}{4} \qquad (\times 4)$$

$$4 \times \textit{tan}(54°) = height$$

$$5.5 = height$$

Now that we know both the "***base***" and the "***height***" of the larger triangle, we can work out the area.

TRIANGLE AREA $= \frac{1}{2} \times \mathbf{8} \times \mathbf{5.5} = \mathbf{22}\text{cm}^2$

Now for the area of the whole Pentagon, as there are 5 larger triangles, we just multiply **22cm**^{2} by **5**.

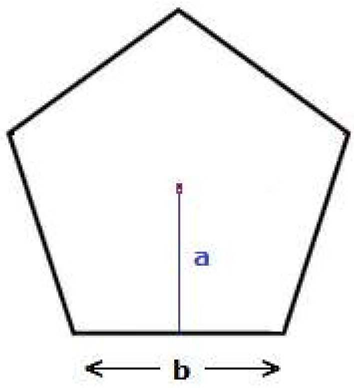
AREA OF PENTAGON $= \mathbf{22}\text{cm}^2 \times \mathbf{5} = \mathbf{110}\text{cm}^2$

What we really needed to know

So all we actually needed, to work out the area of 1 of the 5 triangles, was the length of the blue line, and the length of a side/edge of the Pentagon.

This blue line we've been using for the height of the triangles. From the center of an edge/side, to the center of the Pentagon, isn't just the height of the triangles.

It's the specific line that is called the ***apothem*** of the Pentagon.



Area of a Pentagon was: (AREA of 1 triangle) × **5**

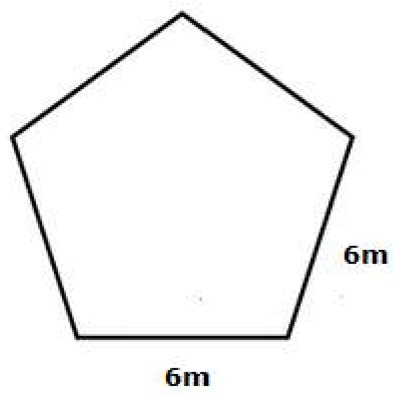
AREA of 1 triangle = $\frac{1}{2}$ × **a** × **b**,

($\frac{1}{2}$ × **a** × **b**) × **5** = $\frac{5}{2}$**ab**

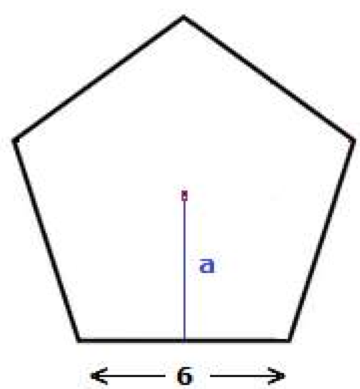
This is how the formula for the area of a regular Pentagon comes about, provided you know **a** and **b**.

Area of a Pentagon Example

(1.1)
Find the area of a Pentagon with the following measurements.



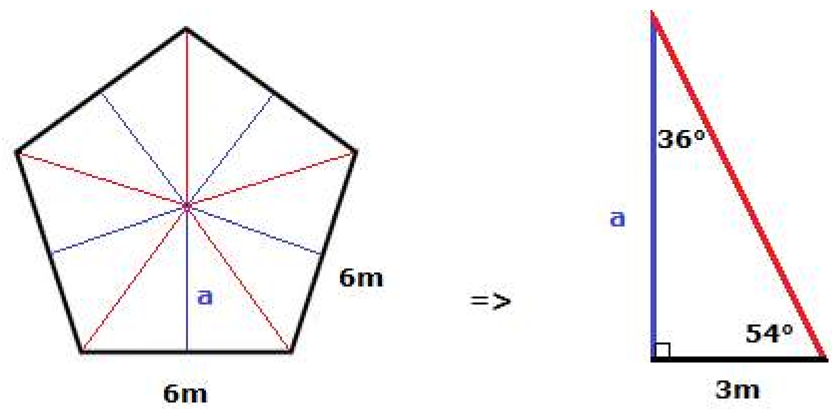
Solution



For using formula $\frac{5}{2}$**ab**,

b = **6** ,   then just need to establish the value of **a**.

The same approach as before with an appropriate Right Angle Triangle can be used.



$tan(54°) = \frac{a}{3}$ (× **3**)

$3 \times tan(54°) = a$

$4.13 = a$

Now: $\frac{5}{2}ab = \frac{5}{2} \times 4.13 \times 6 = 61.95$

The area of the Pentagon is **61.95**m^2.