

Integral Calculus Formula Sheet

Derivative Rules:

$\frac{d}{dx}(c) = 0$	$\frac{d}{dx}(x^n) = nx^{n-1}$	
$\frac{d}{dx}(\sin x) = \cos x$ $\frac{d}{dx}(\sec x) = \sec x \tan x$ $\frac{d}{dx}(\tan x) = \sec^2 x$	$\frac{d}{dx}(\cos x) = -\sin x$ $\frac{d}{dx}(\csc x) = -\csc x \cot x$ $\frac{d}{dx}(\cot x) = -\csc^2 x$	$\frac{d}{dx}(a^x) = a^x \ln a$ $\frac{d}{dx}(e^x) = e^x$
$\frac{d}{dx}(cf(x)) = c \frac{d}{dx}(f(x))$	$\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$	
$(f \cdot g)' = f' \cdot g + f \cdot g'$	$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$	$\frac{d}{dx}(f(g(x))) = f'(g(x))g'(x)$

Properties of Integrals:

$\int kf(u)du = k \int f(u)du$	$\int [f(u) \pm g(u)]du = \int f(u)du \pm \int g(u)du$
$\int_a^a f(x)dx = 0$	$\int_a^b f(x)dx = -\int_b^a f(x)dx$
$\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$	$f_{ave} = \frac{1}{b-a} \int_a^b f(x)dx$
$\int_{-a}^a f(x)dx = 2 \int_0^a f(x)dx$ if f(x) is even	$\int_{-a}^a f(x)dx = 0$ if f(x) is odd
$\int_a^b g(f(x))f'(x)dx = \int_{f(a)}^{f(b)} g(u)du$	$\int udv = uv - \int vdu$

Integration Rules:

$\int du = u + C$ $\int u^n du = \frac{u^{n+1}}{n+1} + C$ $\int \frac{du}{u} = \ln u + C$ $\int e^u du = e^u + C$ $\int a^u du = \frac{1}{\ln a} a^u + C$	$\int \sin u du = -\cos u + C$ $\int \cos u du = \sin u + C$ $\int \sec^2 u du = \tan u + C$ $\int \csc^2 u du = -\cot u + C$ $\int \csc u \cot u du = -\csc u + C$ $\int \sec u \tan u du = \sec u + C$	$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan\left(\frac{u}{a}\right) + C$ $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin\left(\frac{u}{a}\right) + C$ $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arcsec}\left(\frac{ u }{a}\right) + C$
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Fundamental Theorem of Calculus:

$F'(x) = \frac{d}{dx} \int_a^x f(t) dt = f(x)$ where $f(t)$ is a continuous function on $[a, x]$.
$\int_a^b f(x) dx = F(b) - F(a)$, where $F(x)$ is <u>any</u> antiderivative of $f(x)$.

Riemann Sums:

$\sum_{i=1}^n ca_i = c \sum_{i=1}^n a_i$ $\sum_{i=1}^n a_i + b_i = \sum_{i=1}^n a_i + \sum_{i=1}^n b_i$	$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(a + i\Delta x) \Delta x$ $\Delta x = \frac{b-a}{n}$
$\sum_{i=1}^n 1 = n$ $\sum_{i=1}^n i = \frac{n(n+1)}{2}$ $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$ $\sum_{i=1}^n i^3 = \left[\frac{n(n+1)}{2} \right]^2$	$\sum_i (\text{height of } i\text{th rectangle}) \cdot (\text{width of } i\text{th rectangle})$ <p><u>Right Endpoint Rule:</u></p> $\sum_{i=1}^n f(a + i\Delta x) (\Delta x) = \sum_{i=1}^n \left(\frac{b-a}{n} \right) f\left(a + i \frac{b-a}{n}\right)$ <p><u>Left Endpoint Rule:</u></p> $\sum_{i=1}^n f(a + (i-1)\Delta x) (\Delta x) = \sum_{i=1}^n \left(\frac{b-a}{n} \right) f\left(a + (i-1) \frac{b-a}{n}\right)$ <p><u>Midpoint Rule:</u></p> $\sum_{i=1}^n f\left(a + \left(\frac{(i-1)+i}{2}\right) \Delta x\right) (\Delta x) = \sum_{i=1}^n \left(\frac{b-a}{n} \right) f\left(a + \left(\frac{(i-1)+i}{2}\right) \frac{b-a}{n}\right)$

Net Change:

Displacement: $\int_a^b v(x) dx$	Distance Traveled: $\int_a^b v(x) dx$	$s(t) = s(0) + \int_0^t v(x) dx$	$Q(t) = Q(0) + \int_0^t Q'(x) dx$
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Trig Formulas:

$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$	$\tan x = \frac{\sin x}{\cos x}$	$\sec x = \frac{1}{\cos x}$	$\cos(-x) = \cos(x)$	$\sin^2(x) + \cos^2(x) = 1$
$\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$	$\cot x = \frac{\cos x}{\sin x}$	$\csc x = \frac{1}{\sin x}$	$\sin(-x) = -\sin(x)$	$\tan^2(x) + 1 = \sec^2(x)$

Geometry Formulas:

<u>Area of a Square:</u> $A = s^2$	<u>Area of a Triangle:</u> $A = \frac{1}{2}bh$	<u>Area of an Equilateral Triangle:</u> $A = \frac{\sqrt{3}}{4}s^2$	<u>Area of a Circle:</u> $A = \pi r^2$	<u>Area of a Rectangle:</u> $A = bh$
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Areas and Volumes:

<p><u>Area in terms of x (vertical rectangles):</u></p> $\int_a^b (\text{top} - \text{bottom}) dx$	<p><u>Area in terms of y (horizontal rectangles):</u></p> $\int_c^d (\text{right} - \text{left}) dy$
<p><u>General Volumes by Slicing:</u> Given: Base and shape of Cross-sections</p> $V = \int_a^b A(x) dx \text{ if slices are vertical}$ $V = \int_c^d A(y) dy \text{ if slices are horizontal}$	<p><u>Disk Method:</u> For volumes of revolution laying on the axis with slices perpendicular to the axis</p> $V = \int_a^b \pi [R(x)]^2 dx \text{ if slices are vertical}$ $V = \int_c^d \pi [R(y)]^2 dy \text{ if slices are horizontal}$
<p><u>Washer Method:</u> For volumes of revolution not laying on the axis with slices perpendicular to the axis</p> $V = \int_a^b \pi [R(x)]^2 - \pi [r(x)]^2 dx \text{ if slices are vertical}$ $V = \int_c^d \pi [R(y)]^2 - \pi [r(y)]^2 dy \text{ if slices are horizontal}$	<p><u>Shell Method:</u> For volumes of revolution with slices parallel to the axis</p> $V = \int_a^b 2\pi rh dx \text{ if slices are vertical}$ $V = \int_c^d 2\pi rh dy \text{ if slices are horizontal}$

Physical Applications:

Physics Formulas	Associated Calculus Problems
<p><u>Mass:</u> Mass = Density * Volume <i>(for 3-D objects)</i> Mass = Density * Area <i>(for 2-D objects)</i> Mass = Density * Length <i>(for 1-D objects)</i></p>	<p><u>Mass of a one-dimensional object with variable linear density:</u></p> $\text{Mass} = \int_a^b (\text{linear density}) \underbrace{dx}_{\text{distance}} = \int_a^b \rho(x) dx$
<p><u>Work:</u> Work = Force * Distance Work = Mass * Gravity * Distance Work = Volume * Density * Gravity * Distance</p>	<p><u>Work to stretch or compress a spring (force varies):</u></p> $\text{Work} = \int_a^b (\text{force}) dx = \int_a^b F(x) dx = \int_a^b \underbrace{kx}_{\text{Hooke's Law for springs}} dx$ <p><u>Work to lift liquid:</u></p> $\text{Work} = \int_c^d (\text{gravity})(\text{density})(\text{distance}) \underbrace{(\text{area of a slice}) dy}_{\text{volume}}$ $W = \int_c^d 9.8 * \rho * A(y) * (a - y) dy \text{ (in metric)}$
<p><u>Force/Pressure:</u> Force = Pressure * Area Pressure = Density * Gravity * Depth</p>	<p><u>Force of water pressure on a vertical surface:</u></p> $\text{Force} = \int_c^d (\text{gravity})(\text{density})(\text{depth}) \underbrace{(\text{width}) dy}_{\text{area}}$ $F = \int_c^d 9.8 * \rho * (a - y) * w(y) dy \text{ (in metric)}$

Integration by Parts:

Knowing which function to call u and which to call dv takes some practice. Here is a general guide:

u	Inverse Trig Function	($\sin^{-1} x, \arccos x, \text{etc}$)
↑	Logarithmic Functions	($\log 3x, \ln(x+1), \text{etc}$)
↕	Algebraic Functions	($x^3, x+5, 1/x, \text{etc}$)
↓	Trig Functions	($\sin(5x), \tan(x), \text{etc}$)
dv	Exponential Functions	($e^{3x}, 5^{3x}, \text{etc}$)

Functions that appear at the top of the list are more like to be u , functions at the bottom of the list are more like to be dv .

Trig Integrals:

Integrals involving $\sin(x)$ and $\cos(x)$:	Integrals involving $\sec(x)$ and $\tan(x)$:
1. If the power of the sine is odd and positive: Goal: $u = \cos x$ i. Save a $du = \sin(x)dx$ ii. Convert the remaining factors to $\cos(x)$ (using $\sin^2 x = 1 - \cos^2 x$.)	1. If the power of $\sec(x)$ is even and positive: Goal: $u = \tan x$ i. Save a $du = \sec^2(x)dx$ ii. Convert the remaining factors to $\tan(x)$ (using $\sec^2 x = 1 + \tan^2 x$.)
2. If the power of the cosine is odd and positive: Goal: $u = \sin x$ i. Save a $du = \cos(x)dx$ ii. Convert the remaining factors to $\sin(x)$ (using $\cos^2 x = 1 - \sin^2 x$.)	2. If the power of $\tan(x)$ is odd and positive: Goal: $u = \sec(x)$ i. Save a $du = \sec(x)\tan(x)dx$ ii. Convert the remaining factors to $\sec(x)$ (using $\sec^2 x - 1 = \tan^2 x$.)
3. If both $\sin(x)$ and $\cos(x)$ have even powers: Use the half angle identities: i. $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$ ii. $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$	<ul style="list-style-type: none"> If there are no $\sec(x)$ factors and the power of $\tan(x)$ is even and positive, use $\sec^2 x - 1 = \tan^2 x$ to convert one $\tan^2 x$ to $\sec^2 x$ Rules for $\sec(x)$ and $\tan(x)$ also work for $\csc(x)$ and $\cot(x)$ with appropriate negative signs
<i>If nothing else works, convert everything to sines and cosines.</i>	

Trig Substitution:

Expression	Substitution	Domain	Simplification
$\sqrt{a^2 - u^2}$	$u = a \sin \theta$	$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$	$\sqrt{a^2 - u^2} = a \cos \theta$
$\sqrt{a^2 + u^2}$	$u = a \tan \theta$	$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$	$\sqrt{a^2 + u^2} = a \sec \theta$
$\sqrt{u^2 - a^2}$	$u = a \sec \theta$	$0 \leq \theta \leq \pi, \theta \neq \frac{\pi}{2}$	$\sqrt{u^2 - a^2} = a \tan \theta$

Partial Fractions:

Linear factors:	Irreducible quadratic factors:
$\frac{P(x)}{(x-r_1)^m} = \frac{A}{(x-r_1)} + \frac{B}{(x-r_1)^2} + \dots + \frac{Y}{(x-r_1)^{m-1}} + \frac{Z}{(x-r_1)^m}$	$\frac{P(x)}{(x^2+r_1)^m} = \frac{Ax+B}{(x^2+r_1)} + \frac{Cx+D}{(x^2+r_1)^2} + \dots + \frac{Wx+X}{(x^2+r_1)^{m-1}} + \frac{Yx+Z}{(x^2+r_1)^m}$
<i>If the fraction has multiple factors in the denominator, we just add the decompositions.</i>	